Simulation of time-discrete stochastic Markov processes through quantum systems using IBM-Qiskit

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Introduction

A stochastic Markov process ("Markov chain") is a succession of observations that can be in a finite number of states, and the probability of each result depends only on the previous observation. First defined in 1907, there has been many developments around this type of stochastic process and their applications in real-world problems from many areas of knowledge.

Quantum systems always have been conceived as simulators of highly complex phenomena due to their exponential power of representation and their inherent randomness. Also, classical systems have several limitations when simulating stochastic processes; quantum systems can achieve significant improvements in the amount of information required and processing time.

Methodology

1. One qubit per Timestep: the state of the nth qubit stores all the probabilities in the nth stage of the process through controlled rotations

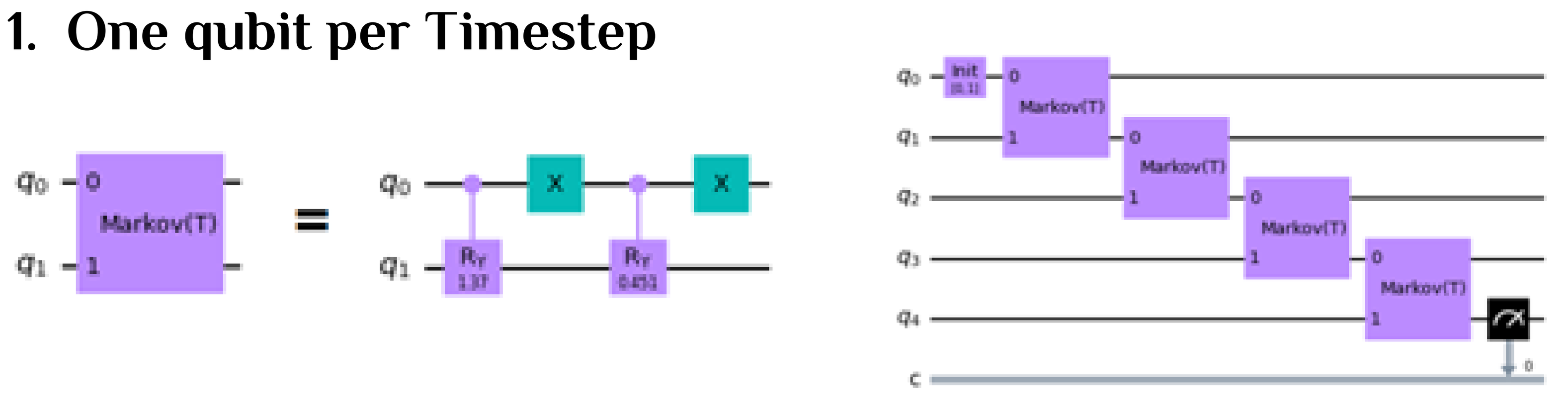
2. One qubit per Sample: one quantum register encodes the previous observation as a control, and in the other each qubit represents a possible outcome of the process

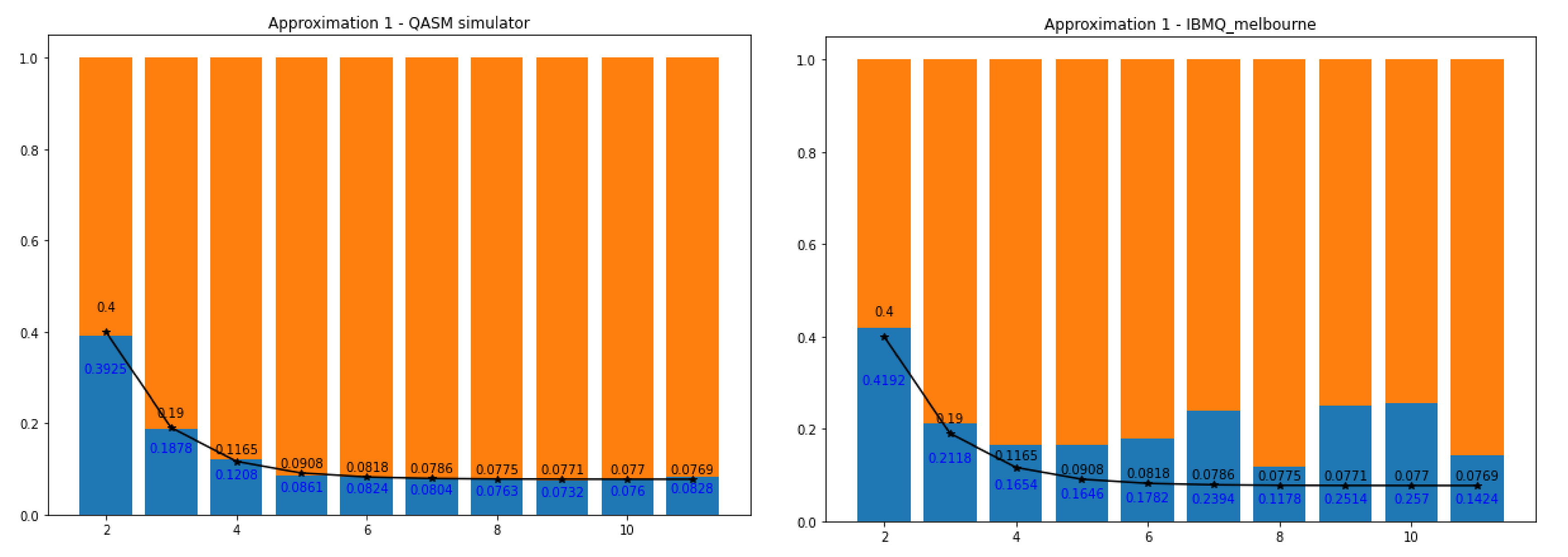
3. Generalization: based on the previous circuits, to build a more general circuit that allows simulating processes with three or four possible outcomes

Results (2x2 matrix)

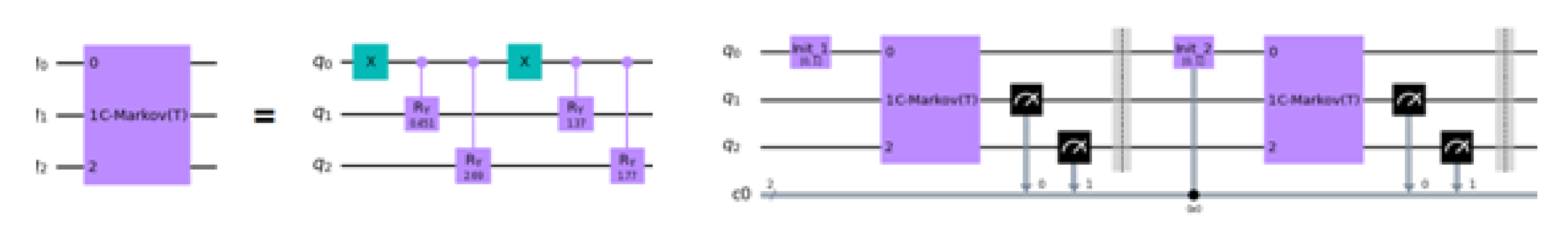
Transition matrix and initial observation used for tests $T = \begin{bmatrix} 0.95 & 0.05 \\ 0.60 & 0.40 \end{bmatrix}$ $E_0 = \begin{bmatrix} 0 & 1 \end{bmatrix}$

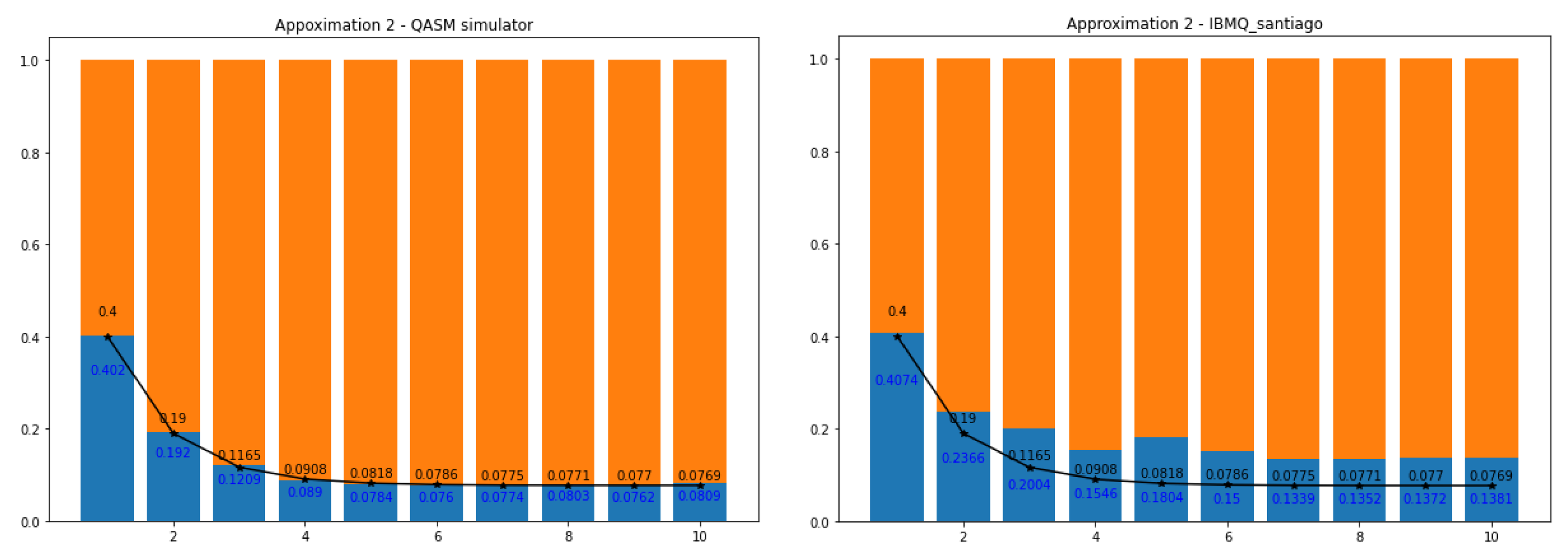
1. One qubit per Timestep



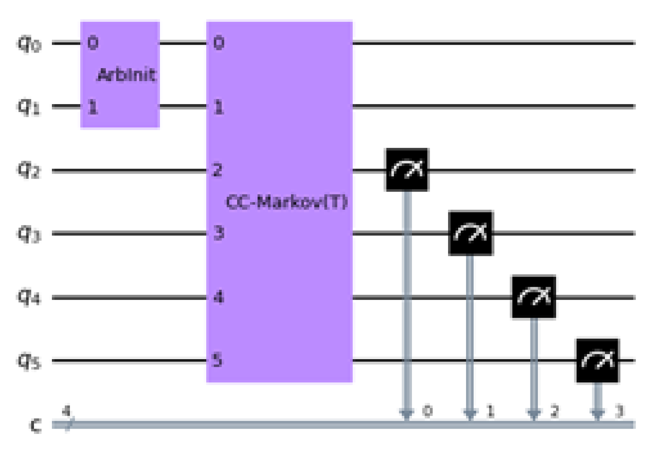


2. One qubit per Sample





Results (3x3 and 4x4 matrices)

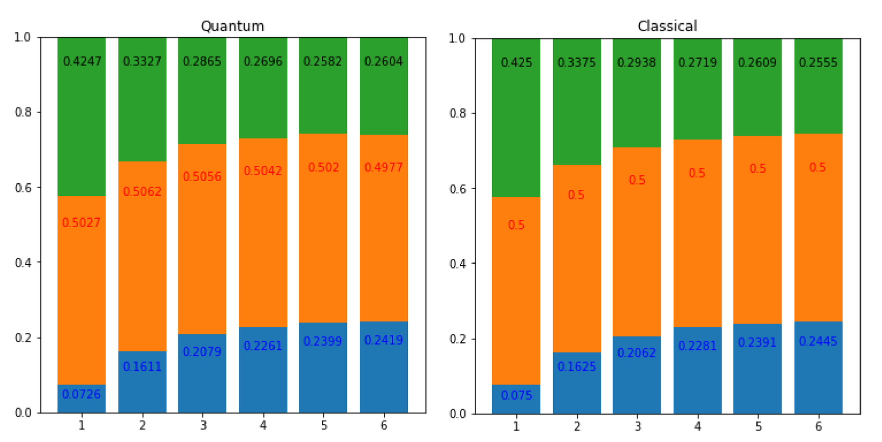


Proposed circuit

Experiment 1:
3x3 matrix for basic gene modeling

$$T = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.5 & 0.5 \end{bmatrix}$$

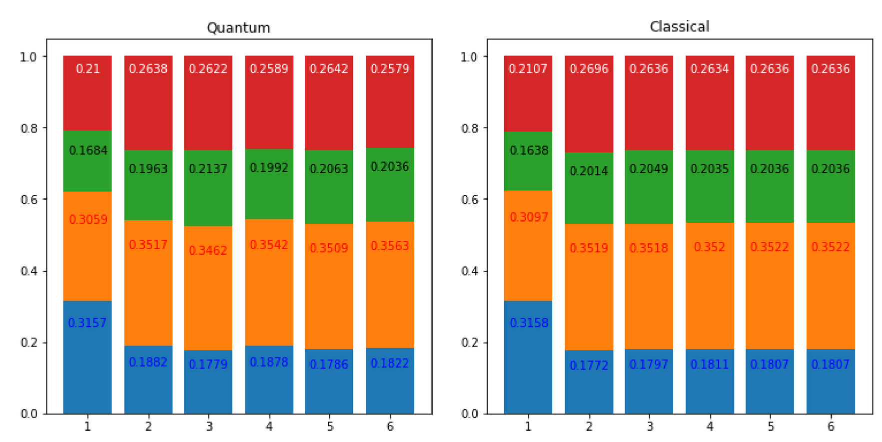
$$E_0 = \begin{bmatrix} 0.1 & 0.1 & 0.8 \end{bmatrix}$$



Experiment 2:
4x4 matrix from a healthcare model

$$T = \begin{bmatrix} 0.17 & 0.33 & 0.21 & 0.29 \\ 0.11 & 0.48 & 0.09 & 0.32 \\ 0.43 & 0.27 & 0.14 & 0.16 \\ 0.09 & 0.26 & 0.40 & 0.25 \end{bmatrix}$$

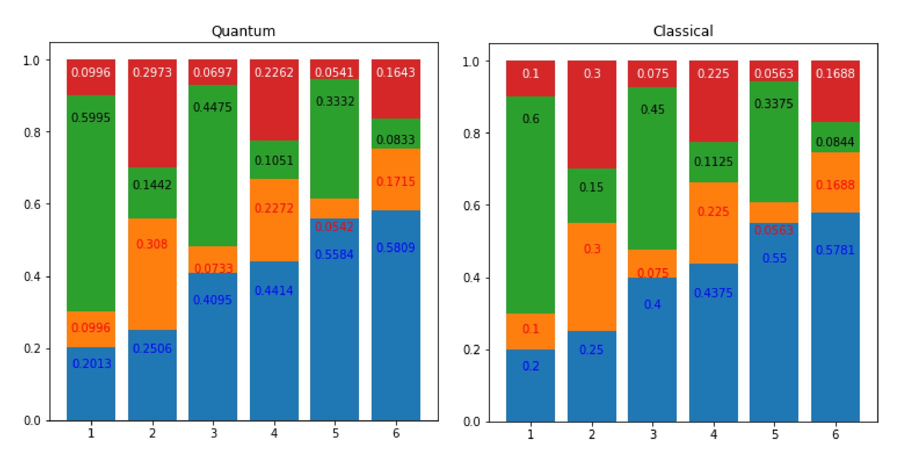
$$E_0 = \begin{bmatrix} 0.15 & 0.15 & 0.62 & 0.8 \end{bmatrix}$$



Experiment 3:
one-dimensional random walk with absorbent state

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$E_0 = \begin{bmatrix} 0 & 0.4 & 0.2 & 0.4 \end{bmatrix}$$



Complexity

n : *number of samples*
t : *number of timesteps*

	Classic	Approx. 1	Approx. 2
Time / Depth	$O(n^2 t)$	$O(nt)$	$O(n^2 t)$
Space / Width	$O(n^2 + nt)$	$O(t \log_2 n)$	$O(n + \log_2 n)$

Further work

- To design and implement more general circuits that allow simulating Markovian processes with larger sample spaces.

- To include both classical and quantum error correction strategies in order to improve the precision of these circuits.

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Link to the video: https://youtu.be/Quzs3KTkN0E